

$$x(t) y(t) \xrightarrow{\mathcal{F}} \frac{1}{2\pi} X(\omega) * Y(\omega)$$

$$Z(\omega) = X(\omega) * Y(\omega) = \int_{-\infty}^{\infty} X(\Omega) Y(\omega - \Omega) d\Omega$$

$$z(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z(\omega) e^{j\omega t} d\omega$$

$$z(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} X(\Omega) Y(\omega - \Omega) d\Omega \right] e^{j\omega t} d\omega$$

$$= \int_{-\infty}^{\infty} X(\Omega) \underbrace{\left[\frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega - \Omega) e^{j\omega t} d\omega \right]}_{\mathcal{F}_\omega^{-1}\{Y(\omega - \Omega)\}} d\Omega$$

$$= \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} y(t) d\Omega$$

$$= 2\pi \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega \right) y(t)$$

$$= 2\pi x(t) \cdot y(t)$$

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